The Futility of Rounding

What do the numbers 445 and 445 have in common? After rubbing your eyes a little, you might decide, quite a lot. Well, here's one way they differ.

<u>Problem 1</u>   Round 445 correct to 1 sf (significant figure). The answer is 400.

<u>Problem 2</u>   Round 445 correct to 2 sf, then round the result correct to 1 sf. The answer is 500.

Well ok, it's not the numbers that differ here, but the method. But as mathematicians, this simple demonstration should make us feel uncomfortable. Depending on how we do the rounding, we end up with two different answers to the same mathematical problem.

So, just stick to the first method - it's a lot easier. Unfortunately, the first method is really for "demonstration" purposes only. As soon as the problem calls for a little calculation, we effectively resort to the second method.

But surely this just a contrived example, designed to confuse students and stir up trouble in the school curriculum. Well, depending which method you use, 4445 rounds to 4000 or 5000, as does 4499, as does any other number between 4445 and 4499. In addition, inconsistency does not vanish on rounding to a higher number of significant figures.

But is this important, or even interesting? When rounding is routinely used to award (and deduct) accuracy marks in mathematics examinations, some students might argue that the anomaly is important, and not a little unfair. But it is also unsatisfactory from another perspective.

Generally in mathematics, it doesn't matter how we go about solving a problem, provided we stick to mathematically legitimate techniques, the different methods will arrive at the same answer. We are free to choose whichever method we prefer or even create a new method. For example, to solve a quadratic equation we might elect to use a factorisation method. Or, if we don't like the look of the coefficients, we might apply the quadratic formula directly. Or we might just guess and check until we find a solution. We do not need to worry about such questions as, which is the "right" method. It all works out the same in the end.

But when different methods yield different results, then mathematics is in trouble. We might even end up arguing over which method to use from the perspective of which answer we most prefer. As a breed, mathematicians tend to be suspicious of disciplines where the method of choice depends on the answer of preference.

Now you might well be tempted to think a little outside regulation is what's needed here to help mathematicians set their house in order. Merely insist everyone uses the simpler one-step method method. That answer can then be declared correct, all others wrong. Such rationalising regulations could well prove popular with teachers, not to mention the lucrative industry of public examinations. The UK now spends an average of £2000 per student per year on examinations - the highest in the world - and probably a similar additional amount preparing students. The regulatory boards would be advised to tread a little warily though, and avoid embarrassing gaffes like the attempt in 1897 by the Indiana General Assembly to legally enforce the value of π to 16/5, rather than the obscure European value of 3.141592654…

Unfortunately, it turns out mathematicians tend to use the multi-step rounding method in calculations, rounding intermediate results to a "sufficiently high" accuracy to avoid handling long numbers, only rounding to the requested accuracy in the final step. However, this has the result that diverging paths can arise at any step in a calculation, wherever intermediate results are combined arithmetically and then rounded. Different routes to the solution can therefore lead to a number of distinct results, depending on how many times the chosen method of solution wanders into the 444 – 499 abyss, or its relatives.

Historically, rounding dates to a time when calculation was performed longhand and it was necessary to compromise between accuracy of the final result and time spent producing it. The advent of cheap powerful calculators with 10 digit displays has rendered rounding pointless. The important and interesting question of how arithmetic error propagates through a calculation remains not only unanswered but completely obscured by rounding.

What rounding selects for in a student is the ability to jump unquestioningly through hoops - flawed hoops at that - without an inclination to explore the underlying mathematics. Students who might wish to understand are met with a recipe of ad hoc rules which make no sense and are inconsistent to apply. *Round your answer to an appropriate degree of accuracy* presupposes anyone can unambiguously define an appropriate degree of accuracy. Students are expected to examine any data available in the question and quote their result to the same degree of accuracy. Often the data varies in accuracy, mixtures of 1sf, 2sf and 3sf are common. Also, should we not teach that the accuracy of the answer is necessarily degraded from that of the input data by the calculation process itself – so why are we asking students to quote to the same level?

The method of rounding as practised in schools has become futile. It is boring, pointless, un-mathematical and a throwback to a time when calculators were unknown and unchallenged learning an ideal. Worst of all, it is easier and always at least as accurate, to simply truncate at (n+1) significant digits rather than round at n significant digits. The continued presence of rounding on the school syllabus obstructs any study of the important and interesting question of error propagation. The only thing rounding does seem particularly good for is confusing students and siphoning off marks at examinations. If students ever questioned this practice, examination boards would find their position very hard to defend.