A Dishonest Question?

Everyone knows that answers can be dishonest, but can a question be dishonest? Surely a question is just an invitation to comment - an invitation which can be accepted or declined. How can it be dishonest?

In her puzzle column of the American magazine “Parade” September 1990, Marilyn vos Savant published a puzzle she had received from a reader.

'Suppose you're on a game show and you're given the choice of three doors. Behind one is a car, behind each of the others is a goat. You pick a door, say door A, and the host, who knows what's behind the other doors, opens another door, say B, which has a goat. He then says: "Do you want to switch to door C?" Is it to your advantage to take the switch?'

vos Savant’s answer was “yes” - switching increases to 2/3 the odds of winning the car.

At first glance the puzzle invites a straight forward application of probability theory and it is very tempting to sketch a probability tree diagram and compare the outcomes.

But is this entirely a question in probability? And what does the vague statement

> ***… and the host, who knows what’s behind the other doors, …***

contribute precisely? Presumably, the statement is intended to be relevant or it wouldn’t be there, but what data specifically does it contribute?

Part of the difficulty is the substantial ambiguity in the original statement, which no doubt contributed to the controversy the puzzle and solution aroused when published in the public domain (unlike its long and non-controversial history in the mathematical literature).

Rather than argue endlessly over what the question might be saying, we will set out the explicit strategy with which we assume the host is required to act. We can and should do this since different host strategies result in different conclusions. If this results in a puzzle different from that intended, well, too bad. Perhaps the original question should be more carefully formulated.

We append the following rules to the statement of the game above:

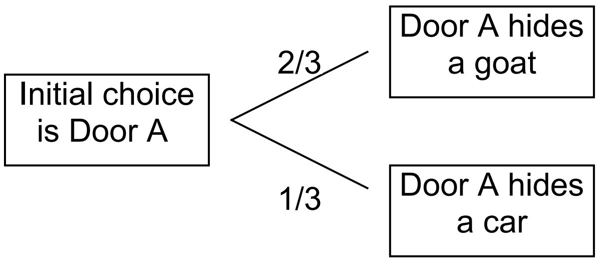
- The host will always invite a contestant to state an initial choice
- The host will then always open a different door <u>and</u> reveal a goat
- The host will then always invite the contestant to open a remaining door

Now in no sense can these rules be seen as deducible from the original statement, and a number of variations are conceivable. We don’t know the host is following <u>any</u> consistent strategy. Perhaps his actions are random. Perhaps on occasion he opens the contestant’s first choice. Or maybe sometimes he decides to reveal the car. Perhaps he follows a number of different strategies depending on whether or not its been a while since anyone won the car. We have no way of knowing from one cited instance. However, as a starting point our appended rules are clear, simple to apply and render the problem unambiguous.

Having fixed the strategy under which the host operates, we now analyse the probability of success for each of three distinctly different contestant strategies, namely where, for the final choice of two doors, the contestant:

1. always sticks
2. randomly reselects
3. always switches

<u>Contestant "always sticks"</u>



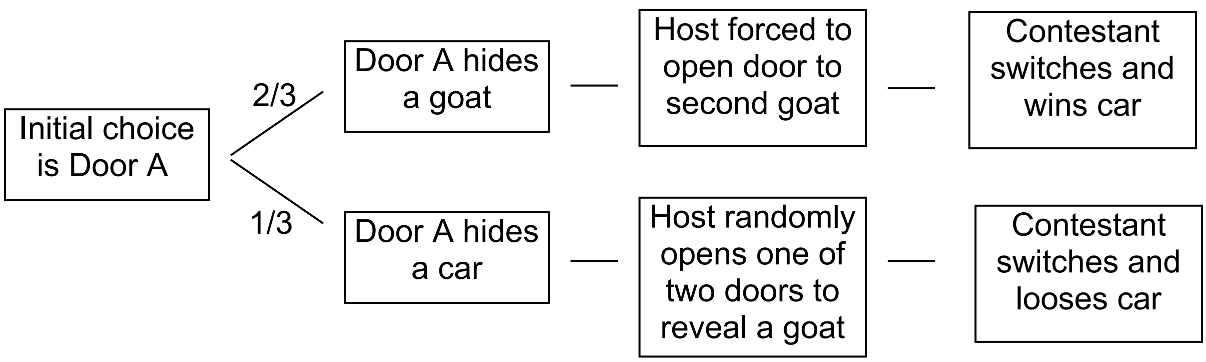
With the initial choice, the contestant's chance of selecting the door with the car is 1/3 and nothing the host says or does subsequently can change this. The contestant's chance of winning the car under the "always stick" strategy is therefore 1/3.

<u>Contestant "randomly reselects"</u>

Under this strategy, when the host reveals the position of one of the goats, the contestant is free to reselect one of the remaining two doors. If he does so at random, the chance of selecting the door with the car is ½. The chance of winning the car under the "randomly reselect" strategy is therefore ½.

<u>Contestant "always switches"</u>

The contestant's initial choice of door can either hide a goat (arising 2/3 of the time) or a car (arising 1/3 of the time).



Where the first choice of door hides a goat, the host is now forced to reveal the whereabouts of the other goat, since under his strategy he cannot under reveal the contestant's first choice <u>nor</u> the car. This eliminates the second goat and, under the "always switch" strategy, the contestant will now choose the door with the car. This scenario occurs 2/3 of the time.

In the second scenario, where the contestant's first choice is the door hiding the car, the "always switch" strategy will always fail. The contestant's total chance of winning the car under the "always switch" strategy therefore remains at 2/3.

To summarise, we see that the contestant's strategies for winning the car compare as

always stick (1/3) < randomly reselect (1/2) < always switch (2/3)

so where the host follows the appended strategy, the contestant's best strategy is to always switch.

What we have done is redefine a badly-posed question of probability theory into a well-posed question in game theory in which the success of various contestant strategies are assessed using probability calculations. The point is that when the question is restated in an unambiguous form, the difficulties disappear and the results become non-controversial. The original question is difficult only in the sense that different people are answering different questions and, not surprisingly, arriving at different conclusions.

With hindsight, we might well ask whether the original question was deliberately ambiguous and provocative. The acid test is surely, does an unambiguous restatement of the question reduce its degree of difficulty? If the only real difficulty is untangling the verbal contortions then the question is of little interest mathematically and we might well be tempted to label it dishonest and distracting. Someone has gone out of their way to increase confusion and argument.

In the end, whether the original ambiguity was intentional (newspapers do tend to flourish on controversy) is perhaps unimportant. Questions will always arise which can be used to surprise, challenge intuition, trick, mislead and belittle. Intuition has a vital role in mathematics just as in any other human endeavour but the moral is, surely, intuition without logic can mislead. Challenging and important problems continually arise in life for which a crucial first step in their solution is the illumination of unsuspected subtleties. This is interesting, instructive and useful and if riddles, teasers and malicious puzzles ignite and fuel an expertise in problem solving, who should begrudge this?