<u>Why the BOM method of factorising works</u>

Given a choice of factorising the trinomials:

$2x^2 + 7x + 6$ $\qquad\qquad$ $y^2 + 7y + 12$

most of us would prefer to tackle the second expression. In fact, under the BOM method, the two expressions are as easy to factorise as each other.

We first demonstrate the method on $2x^2 + 7x + 6$ and then explain why it works in general.

$2x^2 + 7x + 6$

↓

$\frac{1}{2}(2x + \quad)(2x + \quad)$

↓

$\frac{1}{2}(2x + \square)(2x + \square)$

↓

$\frac{1}{2}(2x + \boxed{3})(2x + \boxed{4})$

↓

$(2x + 3)(x + 2)$

Step 1 Replace the **$2x^2$** with (2x)(2x) dividing by 2 to compensate.

Step 2 Now look for two numbers that:

multiply to make 12
and add to make 7

x
$2x^2 + 7x + 6$

They are 3 and 4. Put one in each bracket - it doesn't matter which goes where.

Step 3 That's it! Just extract any common factors, and cancel. Check by multiplying out.

<u>**$2x^2 + 7x + 6$** $= (2x + 3)(x + 2)$</u>

The above demonstration shows how to apply the method but it doesn't explain why the steps lead to the correct answer - it is not a mathematical proof. So why does the BOM method work in general?

Consider the factorisation of the general trinomial $ax^2 + bx + c$

$ax^2 + bx + c = (a^2x^2 + bax + ac)$ 🕑a $= (y^2 + by + ac)$ 🕑a (with y 🕓ax)

Now, $y^2 + by + ac$ factorises if its discriminant $(b^2 – 4ac)$ is a perfect square 4, 9, 16, 25, 36... But $ax^2 + bx + c$ has the same discriminant. So $y^2 + by + ac$ factorises if $ax^2 + bx + c$ factorises. All that remains is to find two numbers which multiply to make ac, and add to make b. Substituting back using y 🕓ax and extracting common factors, yields the required factorised expression in x.

The reader is invited to test the efficacy of the method on $15x^2 - 19x - 8$