Graph Theory and Gender Politics

This article compares the total number of 'encounters' and 'affairs' between two genders of a population, rigorously establishing the following results:

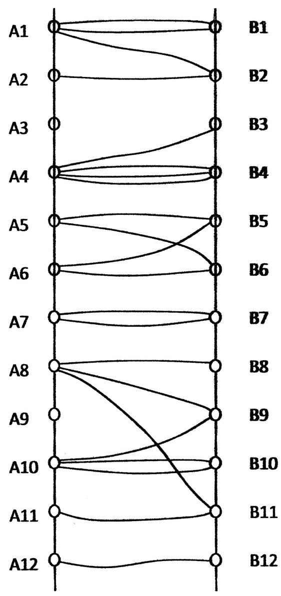
The AB Theorem:

> The total number of encounters summed over gender A is exactly equal to the total number of encounters summed over gender B.
>
> The total number of affairs summed over gender A is exactly equal to the total number of affairs summed over gender B.

Mathematically, the AB Theorem is not especially surprising, though its conclusions do run counter to conventional gender perceptions.

Here we sidestep the definition of social terms such as 'encounters' and 'affairs', proving the AB Theorem using only the fundamental constructions of Graph theory, in which *nodes* of a map are connected to each other by *edges* (see Figure 1). Graph Theory has a long history in analysing connectivity questions in mathematics, such as the Konigsberg Bridge problem, Euler's Formula and the Four Colour Theorem.

Encounters-Affairs Map



Proof:

Label each element of gender A by $A_1, A_2, A_3 \ldots A_n$. Do likewise with $B_1, B_2, B_3 \ldots B_n$ for gender B (see figure 1). We assume the population is composed of equal numbers of the two genders. Later we can relax this assumption without loss of generality.

The theorem will be proved without reference to any sociological preconceptions, such as who in gender A is married to who in gender B, but for ease of visualization we may arrange the elements in marriage order: A_1 is married to B_1, A_2 is married to B_2, A_3 is married to B_3,..., A_m is married to B_m, where m is a number less than or equal to n. Hence each gender consists of m married elements and n-m unmarried elements.

Next, connect elements of gender A with elements of gender B according to their encounters, where each and every encounter generates a new connecting line between the two elements in question. In graph theory, the elements are called *nodes* and the connecting lines are called *edges*. In our completed map (figure 1) there can be any number of edges connecting any two nodes. Of course, in the human society, where the terms 'sexual encounters' and 'love affairs' are in general use, we expect to find a large number of edges connecting two married nodes, with possibly a smaller number of additional edges connecting nodes not married to each other.

Having completed the encounter-affair map, we simply sum the total number of encounters for each gender. We see from the table below, that the total number of encounters summed over the gender A is equal to the total number of encounters summed over gender B.

Encounters-Affairs Table

		Gender A			Gender B	
Element		**Encounters**	**Affairs**		**Encounters**	**Affairs**
1		3	2		2	1
2		1	1		2	2
3		0	0		1	1
4		4	2		3	1
5		2	2		2	2
6		2	2		2	2
7		2	1		2	1
8		3	3		1	1
9		0	0		2	2
10		3	2		2	1
11		1	1		2	2
12		1	1		1	1
Tot		**22**	**17**		**22**	**17**

Presented in this rather graphic manner, the result is perhaps not too surprising. After all, A_i cannot have a encounter with B_j without B_j having a encounter with A_i so the two totals are bound to increment in tandem. More surprising, and controversial perhaps, is that the total number of affairs conducted by each gender must also be identical.

An affair between A_i and B_j is defined to exist when one or more edges exist connecting the nodes A_i and B_j. The number of affairs of any one element A_i is defined as the total number of nodes in gender B connected to A_i by at least one edge. For example, in the diagram above, we see that

A8 generates 3 encounters and 3 affairs
A10 generates 3 encounters and 2 affairs
B12 generates 1 encounter and 1 affair
B9 generates 2 encounters and 2 affairs

Clearly the total number of affairs must be less than or equal to the total number of encounters. Again, referring to the diagram, if we sum the total number of encounters and affairs for each gender, we arrive at the totals listed in the accompanying table. Genders A and B each generate a total of 22 encounters and 17 affairs , despite the seemingly arbitrary way each of these totals can be distributed across the two genders.

Of course, one specific encounter-affair map does not a general result prove. For a complete mathematical proof we need to check the truth of the AB Theorem for *every* possible map. For that we need a little more graph theory.

For maps with n elements to each gender, the number of distinct possible maps is astronomically large, and ranges from maps where zero elements of A connect to zero elements of B (the *null* map), through to maps where every element of A connects to every element of B (the *complete* map).

The null map generates zero encounters and zero affairs for each gender, while the complete map generates (in excess of) n^2 encounters and (precisely) n^2 affairs for each gender. Being in some sense the two extremes of all such maps, these two maps are relatively easy to analyse.

Proving the AB Theorem for *every* encounter-affair map has to proceed indirectly. Suppose we start out from the null map and proceed in increments of a single encounter towards our given map. After one increment from the null map, summing the total encounters and affairs for each gender, we arrive, unsurprisingly, at 1 encounter and 1 affair for each gender.

Suppose now we add a second encounter to our map. Immediately, two distinct possible cases arise: we increase by 1 the number of encounters without increasing the number of affairs, or we increase the number of encounters and the number of affairs both by 1. The important point is that we cannot increase by 1 the number of encounters of one gender, without doing likewise for the other. Nor can we increase by 1 the number of affairs of one gender, without doing likewise for the other gender.

This key observation is all we need to prove the general result, for now we can iterate from the simplest map to the final map, and at each step verify that the total number of encounters and affairs between the two genders increment by the same amount, zero or one. Since the two statistics start out equal for each gender, they remain equal, regardless of how we generate the final map.

Although such terms as encounter and affair are defined consistently and unambiguously in terms of the number of edges in the mathematics of graph theory, the corresponding human analogues 'sexual encounters' and 'love affairs' have not (and perhaps cannot) be consistently and unambiguously defined in the social arena. The AB Theorem stands, without support from, or reference to, any sociological perspectives. Consequently, its interpretation and application in the social arena, remains to be evaluated. However, the AB Theorem does provide a rigorous basis for many further extensions in the field.

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